Simultaneous Motion and Stiffness Control for Soft Pneumatic Manipulators based on a Lagrangian-based Dynamic Model

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Background of Soft Robots

Advantages of Soft Robots

- Adaptability to complex environment
- Perform delicate tasks

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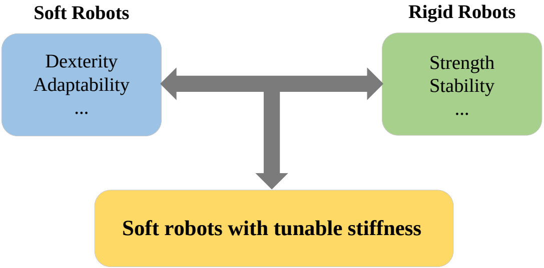
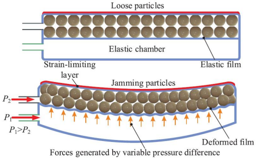
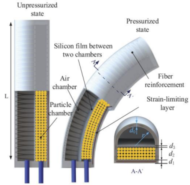


Soft Robots with Particle Jamming Mechanism

- Soft robots with particle jamming mechanism[1−2]





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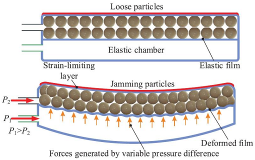
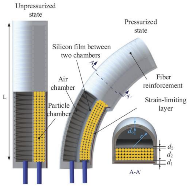
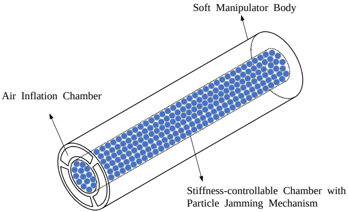




- ***Our Robot:*** 3D pneumatic manipulators with particle jamming backbone



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Continuous Stiffness Control

Previous stiffness-tuning mechanism usually model stiffness as ***discrete states***

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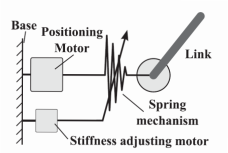
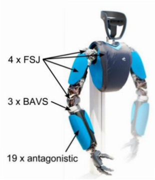
Advantages of continuous stiffness control

- Damping out the unwanted dynamics
- Achieving over-ranging movements

Rigid robots: Variable Stiffness Actuators (VSA) in Hand-Arm system [3]





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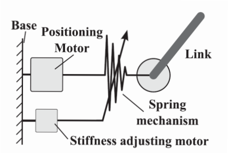

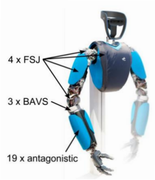

Objective

Simultaneously control the precise motion and continuous stiffness for ***soft pneumatic manipulators***

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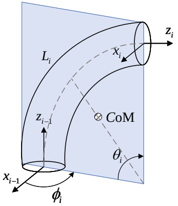
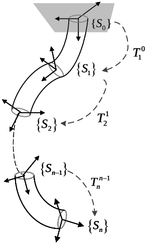
Kinematics based on Piecewise Constant Curvature Assumption



(a)



(b)

- Homogeneous transformation matrix mapping from $\{S_{i-1}\}$ to $\{S_i\}$:

$$T_i^{i-1}(\phi_i,\theta_i) = \begin{bmatrix} C_{\phi_i}^2(C_{\theta_i}-1)+1 & S_{\phi_i}C_{\phi_i}(C_{\theta_i}-1) & C_{\phi_i}S_{\theta_i} & \frac{L_i}{\theta_i}C_{\phi_i}(1-C_{\theta_i}) \\ S_{\phi_i}C_{\phi_i}(C_{\theta_i}-1) & C_{\phi_i}^2(1-C_{\theta_i})+C_{\theta_i} & S_{\phi_i}C_{\theta_i} & \frac{L_i}{\theta_i}S_{\phi_i}(1-C_{\theta_i}) \\ -C_{\phi_i}S_{\theta_i} & -S_{\phi_i}S_{\theta_i} & C_{\theta_i} & S_{\theta_i} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

- Center of Mass (CoM) $p^0_{i\text{-CoM}}$ in the global frame [4]:

$$\begin{bmatrix} p^0_{i\text{-CoM}} \\ 1 \end{bmatrix} = T_1^0(\phi_1,\theta_1)\cdot T_2^1(\phi_2,\theta_2)\cdots T_i^{i-1}(\phi_i,\theta_i)\cdot \begin{bmatrix} p^i_{i\text{-CoM}} \\ 1 \end{bmatrix} \tag{2}$$

[4] R. K. Katzschmann, C. Della Santina, Y. Toshimitsu, A. Bicchi, and D. Rus, "Dynamic motion control of multi-segment soft robots using piecewise constant curvature matched with an augmented rigid body model," in *2019 2nd IEEE International Conference on Soft Robotics (RoboSoft)*. IEEE, 2019, pp. 454–461

Dynamic modeling based on Euler-Lagrange Theory

- Euler-Lagrange equation $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = f$:

$$M(q)\ddot{q} + V(q, \dot{q}) + D(q)\dot{q} + G(q) + Kq = A(q)\tau_A, \tag{3}$$

Matrices are defined in Ref [4].

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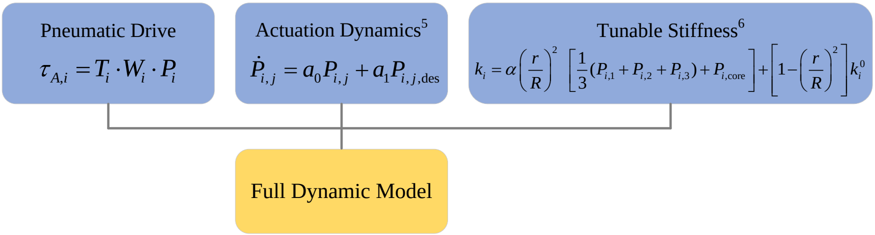
- Adjusting the model from the following aspects:



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Dynamic modeling based on Euler-Lagrange Theory

- ***Full Dynamic Model***

$$
\begin{aligned}
&M(q)\ddot{q} + V(q,\dot{q}) + D(q)\dot{q} + G(q) + \left[1 - \left(\frac{r}{R}\right)^2\right] K^0 q \\
&= A(q) \cdot T \cdot W \cdot P - \left(\frac{r}{R}\right)^2 K_{\text{core}}(P) q
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
\dot{k}_i = \alpha \left(\frac{r}{R}\right)^2 [&\frac{1}{3} a_0 (P_{i,1} + P_{i,2} + P_{i,3}) + b_0 P_{i,\text{core}} \\
&+ \frac{1}{3} a_1 (P_{i,1,\text{des}} + P_{1,2,\text{des}} + P_{i,3,\text{des}}) + b_1 P_{i,\text{core,des}}]
\end{aligned}
\tag{5}
$$

Dynamic modeling based on Euler-Lagrange Theory

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$$
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\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\dot{k}_i &= \alpha\left(\frac{r}{R}\right)^2 [\frac{1}{3}a_0(P_{i,1} + P_{i,2} + P_{i,3}) + b_0 P_{i,\text{core}} \\
&\quad + \frac{1}{3}a_1(P_{i,1,\text{des}} + P_{1,2,\text{des}} + P_{i,3,\text{des}}) + b_1 P_{i,\text{core},\text{des}}]
\end{aligned} \tag{5}
$$

- ***States Differential Equation***

States: $\mathbf{x} = \begin{bmatrix} q & \dot{q} & K_J & P \end{bmatrix}^T$

Inputs: $\mathbf{u} = P_{\text{des}} = \begin{bmatrix} P_{1,\text{des}} & \cdots & P_{i,\text{des}} & \cdots & P_{n,\text{des}} \end{bmatrix}^T$

State space form:

$$
\dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \\ \dot{K}_J \\ \dot{P} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M(q)^{-1}\left(A(q)\cdot T\cdot W\cdot P - \left(\frac{r}{R}\right)^2 K_{core}(P)q - V(q,\dot{q}) - D(q)\dot{q} - G(q) - \left[1-\left(\frac{r}{R}\right)^2\right]K^0 q\right) \\ \begin{bmatrix} \dot{k}_1 & \dots & \dot{k}_i & \dots & \dot{k}_n \end{bmatrix}^T \\ \begin{bmatrix} \dot{P}_{1,1} & \dot{P}_{1,2} & \dot{P}_{1,3} & \dot{P}_{1,\text{core}} & \dots & \dot{P}_{i,j} & \dot{P}_{i,\text{core}} & \dots & \dot{P}_{n,1} & \dot{P}_{n,2} & \dot{P}_{n,3} & \dot{P}_{n,\text{core}} \end{bmatrix}^T \end{bmatrix} \tag{6}
$$

Control in the Configuration Space

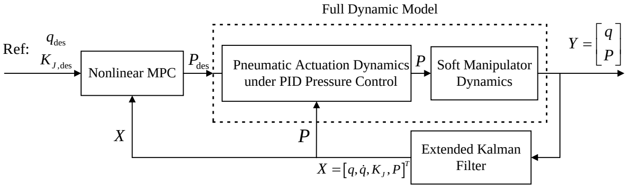


Figure: Proposed NMPC for the soft manipulator with controllable stiffness

Control in the Configuration Space

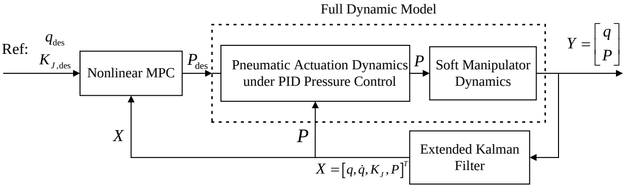


Figure: Proposed NMPC for the soft manipulator with controllable stiffness

- Discrete-time state space: $x_{k+1} = f_{\mathrm{RK4}}(x_k, u_k, \delta t)$

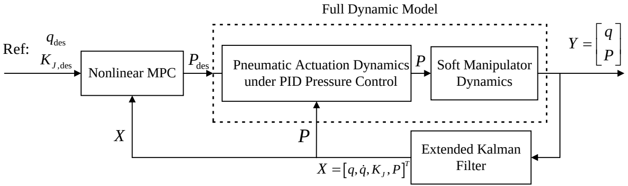
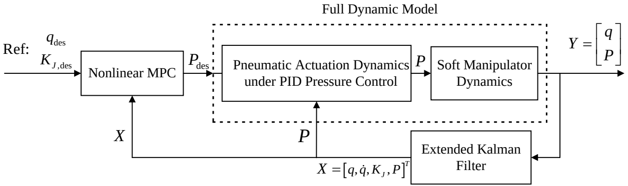


Figure: Proposed NMPC for the soft manipulator with controllable stiffness

- Discrete-time state space: $x_{k+1} = f_{\mathrm{RK4}}(x_k, u_k, \delta t)$
- Nonlinear model predictive control (NMPC) problem formulation:

$$\min_{P_{\mathrm{des}}(k:k+m-1)} \sum_{i=0}^{p-1} (\|q(k+i|k) - q_{\mathrm{des}}(k+i|k)\|^2_{Q_q} + \|K_J(k+i|k) - K_{J,\mathrm{des}}(k+i|k)\|^2_{Q_K}$$
$$+ \|P_{\mathrm{des}}(k+i|k) - P_{\mathrm{des}}(k+i-1|k)\|^2_R) + \|P_{\mathrm{des}}(k+i|k)\|^2_S\,,$$
$$\text{s.t.} \quad x_{k+i+1} = f_{\mathrm{RK4}}(x_{k+i}, u_{k+i}, \delta t), \quad x_0 = x_{\mathrm{init}}\,,$$
$$P_{\min} \leqslant P_{i,j,\mathrm{des}}(k+i) \leqslant P_{\max}\,, \quad V_{\min} \leqslant P_{i,\mathrm{core}}(k+i) \leqslant V_{\max}\,,$$
$$0 \leq i \leq p-1. \tag{7}$$



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- Estimate the full states by using an extended Kalman filter (EKF)

Control in the Task Space

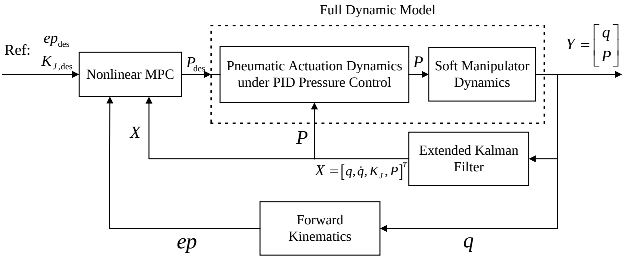


Figure: Extended NMPC enables to track the desired position trajectory in the task space and the stiffness trajectory simultaneously

- Adjusted cost function in the task space:

$$\min_{P_{des}(k:k+m-1)} \sum_{i=0}^{p-1} (\|e(k+i|k) - e_{des}(k+i|k)\|^2_{Q_e} + \|K_J(k+i|k) - K_{J,des}(k+i|k)\|^2_{Q_K}$$

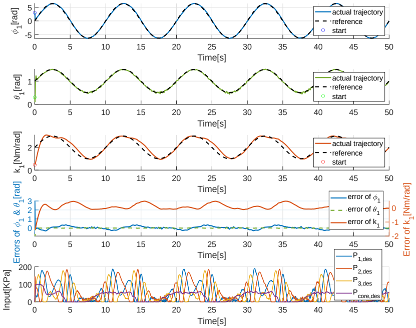
$$+ \|P_{des}(k+i|k) - P_{des}(k+i-1|k)\|^2_R) + \|P_{des}(k+i|k)\|^2_S \,, \quad (8)$$

Results in the Configuration Space

- E.g. One Segment
- The sinusoidal reference trajectories of configuration joint and stiffness:

$$\begin{cases} \phi_{1,\text{des}}(t) = 2\pi \sin(\dfrac{\pi}{5}t) \\ \theta_{1,\text{des}}(t) = 0.5 \sin(\dfrac{\pi}{5}t) + 1 \\ k_{1,\text{des}}(t) = \sin(\dfrac{\pi}{5}t) + 2 \end{cases}$$

- Simulation results in the configuration space



Results in the Task Space

- The circular reference trajectory and the same reference stiffness trajectory:

$$e_{des}(t) = \begin{bmatrix} 0.0716\cos(\varpi t) & 0.0716\sin(\varpi t) & -0.124 \end{bmatrix}^T$$

- Simulation results of tracking slow and quick references in the task space:

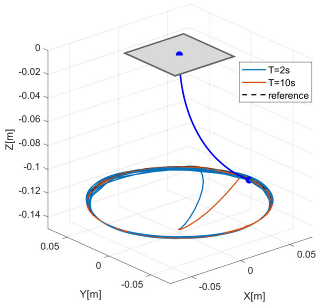


Table: Average and Maximum Euclidean Distance Error of Tracking the Slow and Quick References in the Task Space

	Avg. Euclidean distance error(m)	Max. Euclidean distance error(m)
Slow reference($\varpi = 0.2\pi$)	0.001	0.0052
Quick reference($\varpi = \pi$)	0.0039	0.0090

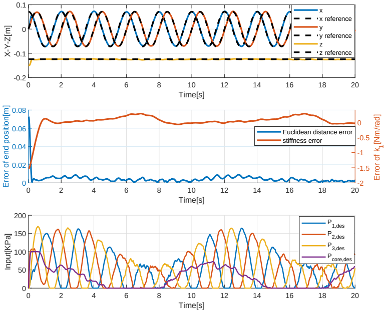


Figure: The performance of the tracking error in the faster case

Conclusion and Future Work

Conclusion:

1. A dynamic model with controllable stiffness
2. NMPC control frameworks to control the motion and stiffness simultaneously
3. The feasibility and robustness of the controllers are validated in the simulation

Future Work:

1. Extend the model with elongation effects
2. The relationship between segment stiffness and Cartesian stiffness
3. Practical application

Acknowledgement:

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Thank You