Simultaneous Motion and Stiffness Control for Soft Pneumatic Manipulators based on a Lagrangian-based Dynamic Model

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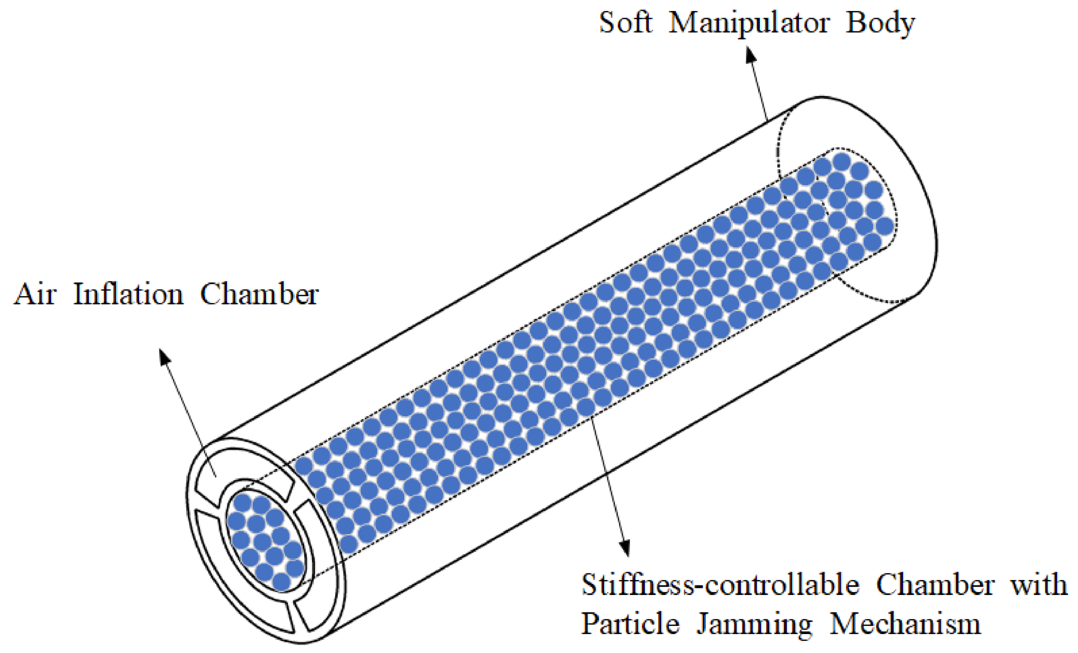
Introduction



Motivation:

- Applications: minimal invasive surgery, picking fruits
- A soft manipulator with tunable stiffness can adapt in unknown environments, and also circumvent the drawbacks of instability and low loading capability
- Continuous stiffness control provides precise motion control with different contact forces, and achieve quick movements from releasing stored energy

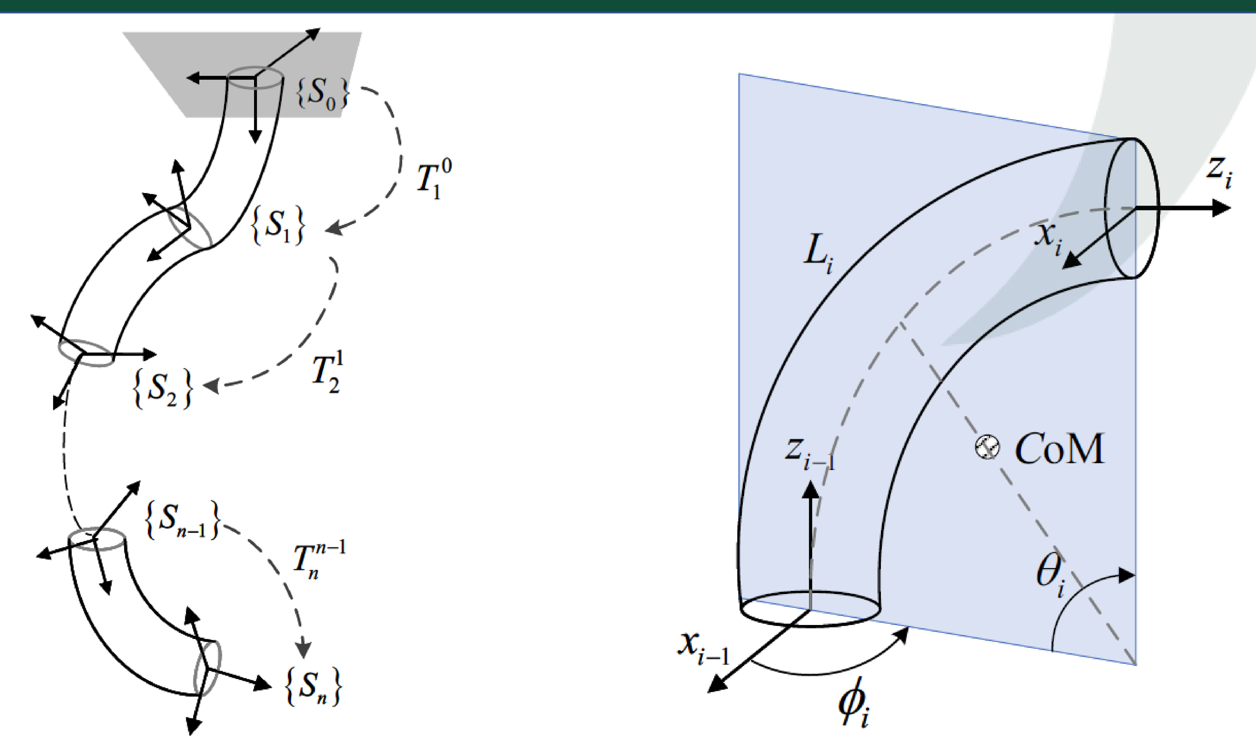
Objective:

- Design stiffness-tunable mechanism for soft manipulators
- Develop a control-oriented dynamic model to capture the high nonlinearity and the strong coupling between actuation and stiffness-tuning
- Control the motion and stiffness of robots simultaneously

Stiffness-tunable Mechanism

- Develop particle jamming mechanism with independent control ability using small vacuum pump

Lagrangian-based Dynamic Model



1) Initial dynamic model:

- Lump each segment into a single mass point in the center of mass (CoM):

$$\begin{bmatrix} p^0_{i\text{-CoM}} \\ 1 \end{bmatrix} = T_1^0(\phi_1,\theta_1)\cdot T_2^1(\phi_2,\theta_2)\cdots T_i^{i-1}(\phi_i,\theta_i)\cdot \begin{bmatrix} p^i_{i\text{-CoM}} \\ 1 \end{bmatrix}$$

- Derive the dynamic equation by the Euler-Lagrange approach:

$$M(q)\ddot{q} + V(q,\dot{q}) + D(q)\dot{q} + G(q) + Kq = A(q)\tau_A$$

2) Full model:

- Considering the pneumatic drive, actuation dynamics and the dynamics of the stiffness-tunable mechanism:

$$M(q)\ddot{q}+V(q,\dot{q})+D(q)\dot{q}+G(q)+\left[1-\left(\frac{r}{R}\right)^2\right]K^0 q$$
$$= A(q)\cdot T\cdot W\cdot P-\left(\frac{r}{R}\right)^2 K_{\text{core}}(P)q$$

3) State dynamics:

- System states and inputs:

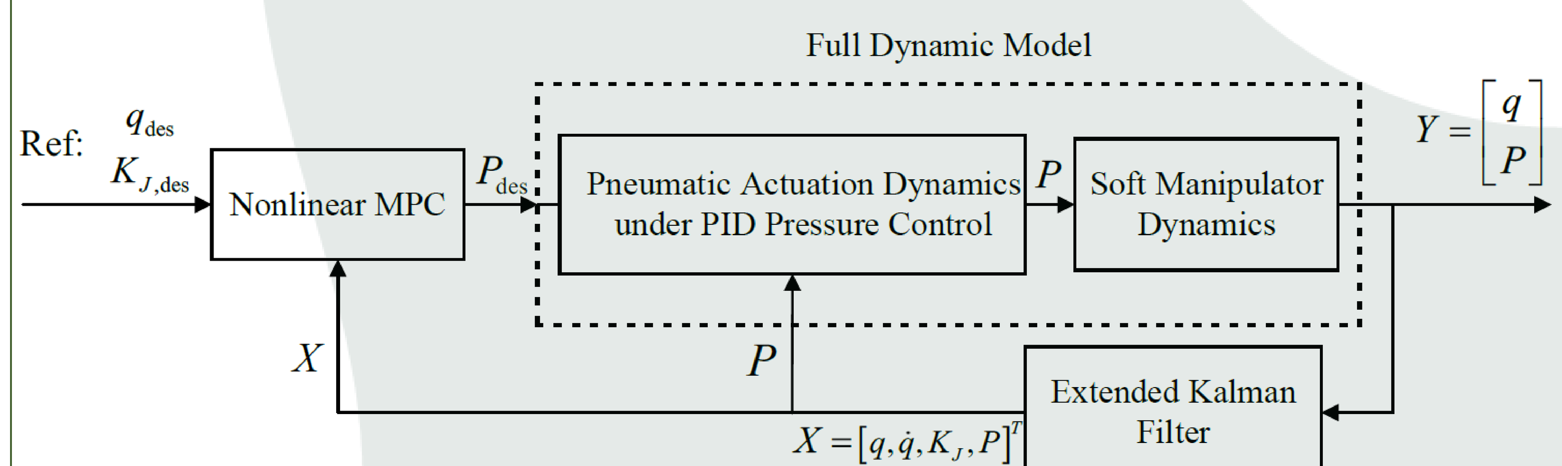
$$\mathbf{x} = [\ q \quad \dot{q} \quad K_J \quad P\ ]^T,$$
$$\mathbf{u} = P_{\text{des}}$$
$$= [\ P_{1,\text{des}} \quad \cdots \quad P_{i,\text{des}} \quad \cdots \quad P_{n,\text{des}}\ ]^T$$

- Full state space formulation:

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \\ \dot{K}_J \\ \dot{P} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M(q)^{-1}\left(A(q)\cdot T\cdot W\cdot P - \left(\frac{r}{R}\right)^2 K_{core}(P)q - V(q,\dot{q}) - D(q)\dot{q} - G(q) - \left[1-\left(\frac{r}{R}\right)^2\right]K^0 q\right) \\ [\ \dot{k}_1 \quad \dots \quad \dot{k}_i \quad \dots \quad \dot{k}_n\ ]^T \\ [\ \dot{P}_{1,1} \quad \dot{P}_{1,2} \quad \dot{P}_{1,3} \quad \dot{P}_{1,\text{core}} \quad \dots \quad \dot{P}_{i,j} \quad \dot{P}_{i,\text{core}} \quad \dots \quad \dot{P}_{n,1} \quad \dot{P}_{n,2} \quad \dot{P}_{n,3} \quad \dot{P}_{n,\text{core}}\ ]^T \end{bmatrix}$$

Nonlinear Model Predictive Controller

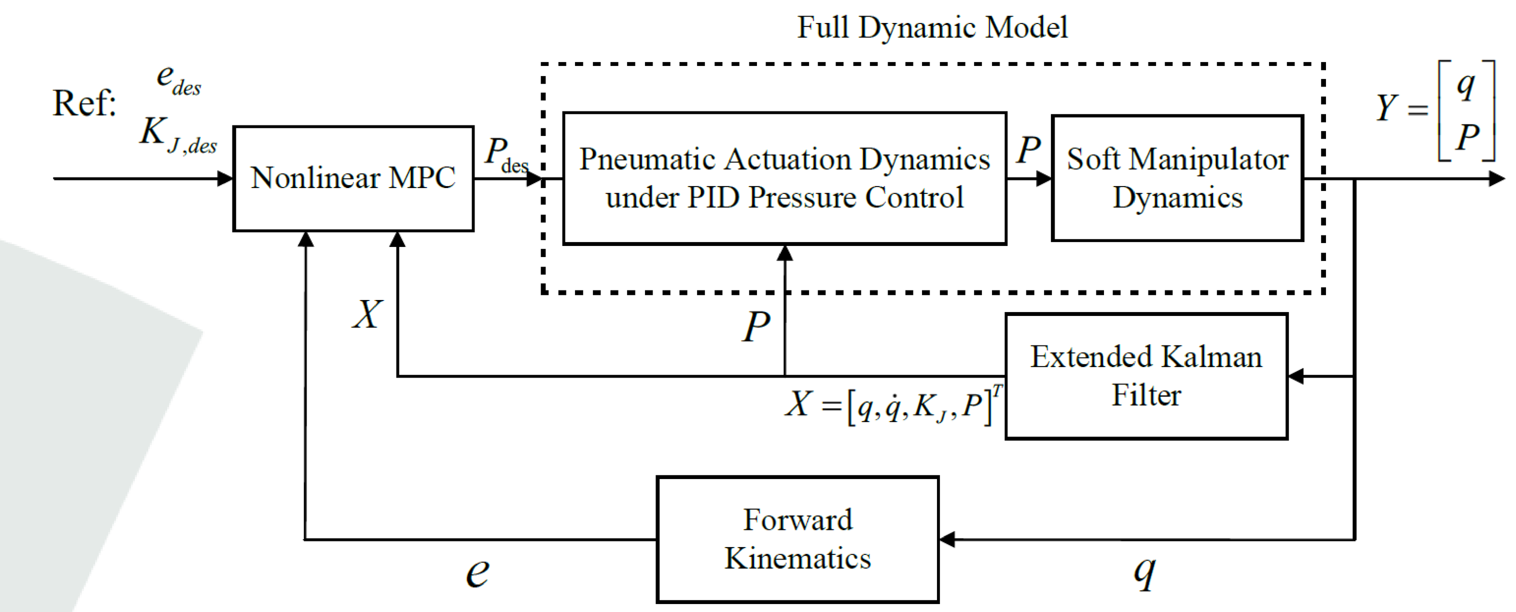
1) Configuration Space



- Cost function under system dynamics and input constraints:

$$\min_{P_{\text{des}}(k:k+m-1)} \sum_{i=0}^{p-1} (\|q(k+i\,|\,k) - q_{\text{des}}(k+i\,|\,k)\|^2_{Q_q}$$
$$+\|K_J(k+i\,|\,k) - K_{J,\text{des}}(k+i\,|\,k)\|^2_{Q_K}$$
$$+\|P_{\text{des}}(k+i\,|\,k) - P_{\text{des}}(k+i-1\,|\,k)\|^2_R)$$
$$+\|P_{\text{des}}(k+i\,|\,k)\|^2_S,$$
$$\text{s.t.}\quad x_{k+i+1} = f_{\text{RK4}}(x_{k+i}, u_{k+i}, \delta t),\quad x_0 = x_{\text{init}},$$
$$P_{\min} \leqslant P_{i,j,\text{des}}(k+i) \leqslant P_{\max},\quad V_{\min} \leqslant P_{i,\text{core}}(k+i) \leqslant V_{\max},$$
$$0 \leq i \leq p-1.$$

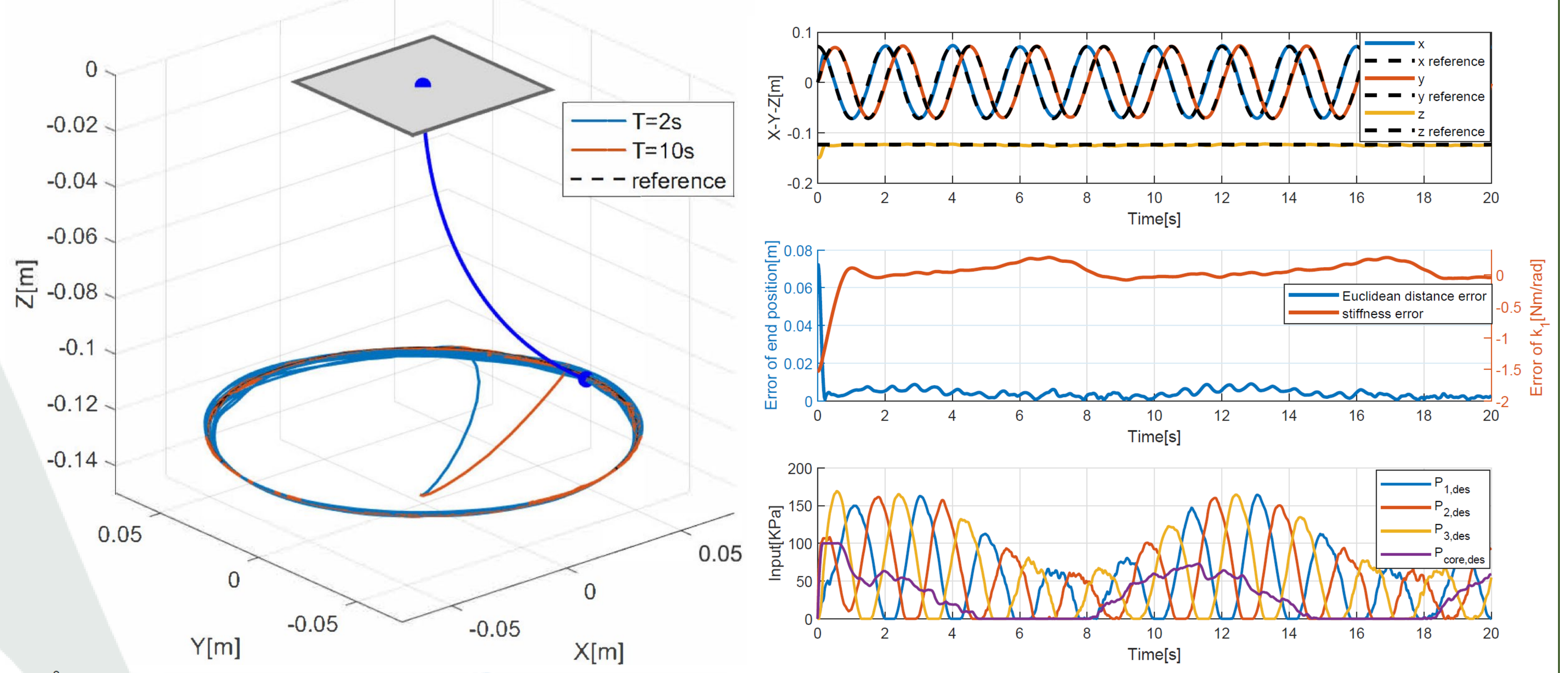
2) Task Space

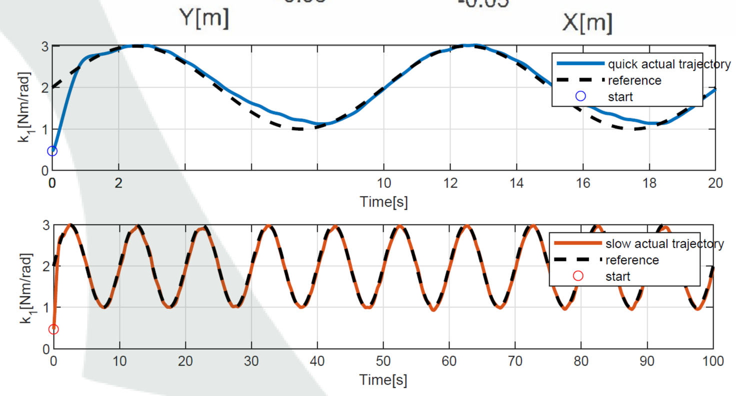


- Adjusted cost function under the same previous constraints:

$$\min_{P_{\text{des}}(k:k+m-1)} \sum_{i=0}^{p-1} (\|e(k+i\,|\,k) - e_{\text{des}}(k+i\,|\,k)\|^2_{Q_e}$$
$$+\|K_J(k+i\,|\,k) - K_{J,\text{des}}(k+i\,|\,k)\|^2_{Q_K}$$
$$+\|P_{\text{des}}(k+i\,|\,k) - P_{\text{des}}(k+i-1\,|\,k)\|^2_R)$$
$$+\|P_{\text{des}}(k+i\,|\,k)\|^2_S$$

Simulation Results





	Avg. Euclidean distance error(m)	Max. Euclidean distance error(m)
Slow reference($\varpi = 0.2\pi$)	0.001	0.0052
Quick reference($\varpi = \pi$)	0.0039	0.0090

- The efficacy of the proposed modeling and controller is validated in the simulation
- For the motion error, the average Euclidean distance error are 0.001m and 0.0039m when tracking slow and quick circular trajectories respectively
- The stiffness error is also minimal when tracking sine wave

Reference

[1] **Y. Mei**, P. Fairchild, V. Srivastava, C. Cao, and X. Tan, "Simultaneous Motion and Stiffness Control for Soft Pneumatic Manipulators based on a Lagrangian-based Dynamic Model," *Proceedings of the 2023 American Control Conference (ACC).*

Acknowledgements

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